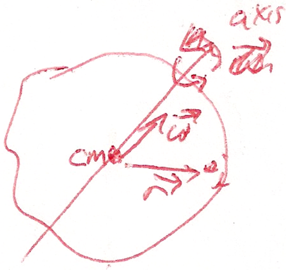
Classical physics



pure rotation

$$\vec{v}_i = \vec{\omega} \times \vec{r}_i$$

$$K = \sum_i \tfrac{1}{2} m_i v_i^2 = \tfrac{1}{2} \sum m_i (\vec{\omega} \times \vec{r}_i) \cdot (\vec{\omega} \times \vec{r}_i)$$

$$= \tfrac{1}{2} \sum_i m_i \overset{A}{\vec{\omega}} \cdot [\overset{B}{\vec{r}_i} \times (\vec{\omega} \times \overset{C}{\vec{r}_i})]$$

$$= \tfrac{1}{2} \vec{\omega} \cdot \sum_i m_i \{ \vec{\omega} (\vec{r}_i \cdot \vec{r}_i) - \vec{r}_i (\vec{r}_i \cdot \vec{\omega}) \}$$

$$= \tfrac{1}{2} \vec{\omega} \cdot \underbrace{\left\{ \sum_i m_i (r_i^2 \underset{\sim}{1} - \vec{r}_i \otimes \vec{r}_i) \right\}}_{\underset{\sim}{I} \text{ a } 3\times 3 \text{ matrix}} \vec{\omega}$$

$$\vec{L} = \underset{\sim}{I} \vec{\omega} \qquad \sum_i m_i \begin{pmatrix} y_i^2 + z_i^2 & -x_i y_i & -x_i z_i \\ -x_i y_i & x_i^2 + z_i^2 & -y_i z_i \\ -x_i z_i & -y_i z_i & x_i^2 + y_i^2 \end{pmatrix}$$

$$\Rightarrow K = \tfrac{1}{2} \vec{L} \cdot \underset{\sim}{I}^{-1} \vec{L}$$

if $\underset{\sim}{I}$ is diagonal, then

$$K = \tfrac{1}{2} \left(\frac{L_x^2}{I_{xx}} + \frac{L_y^2}{I_{yy}} + \frac{L_z^2}{I_{zz}} \right)$$

If $I_{xx} = I_{yy} = I$ and $I_{zz} = I' \neq I$

then $K = \tfrac{1}{2} \left(\dfrac{L_x^2 + L_y^2}{I} + \dfrac{L_z^2}{I'} \right) = \tfrac{1}{2} \left(\dfrac{L^2 - L_z^2}{I} + \dfrac{L_z^2}{I'} \right)$

Quantum physics

~~For a state~~ A States of definite L^2 and L_z are states of definite K:

$$L^2 = L(L+1) \hbar^2 \quad \text{and} \quad L_z = M_L \hbar$$

$$K = \tfrac{1}{2} \left(\frac{L(L+1) - M_L^2}{I} + \frac{M_L^2}{I'} \right)$$

For diatomic molecule, $I' \ll I$ (z-axis is along molecular ~~bond~~)

and so $M_L = 0$ (or else K is very large.

$$\Rightarrow K = \frac{L(L+1)}{2I}$$